Homework 1

due August 31, 2015

[20 pts.]

Name: _____

Step 1: Without using any notes, internet, books, friends, whatever, answer these questions to the best of your ability. You will not be graded on whether you get the right answer - rather, the point of this exercise is to help you determine where you stand with the math needed for this course. ***DO NOT CHEAT!***

Step 2: Grade your answers to step 1 using whatever resource you trust. Write the correct answer in a different color pen/pencil, or on a new sheet of paper.

Step 3: Turn in both your original answers and the graded/corrected answers on the first day of class (Aug. 31, 2015).

Problem 1

Evaluate the following to form an expression in terms of x and t alone:

(a) if $z(t, x) = 4t - \frac{1}{x}$, what is $\frac{\partial}{\partial x}\left(\sqrt{z}\right)$?

(b) if $y(x) = 4x^5$, and $z(x) = \sin(x)$, what is $\frac{d}{dx}\left(4y^2 - 2z\right)$?

Problem 2

For the following problems, expand the expressions so that the del-operator (i.e. ∇) is no longer present. In what follows, $\psi(x, y)$ is a scalar field and $\mathbf{u}(x, y)$ is a 2-dimensional vector with components $\mathbf{u}(x, y) = u(x, y)\hat{\mathbf{i}} + v(x, y)\hat{\mathbf{j}}$ where $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ are the unit vectors in the x- and y-directions, respectively.

(a) $\nabla\psi =$

(b) $\nabla \cdot \mathbf{u} =$

(c) $\nabla \times \mathbf{u} =$

(d) $\nabla \cdot (\psi \mathbf{u}) =$

(e) $\nabla \cdot (\nabla \times \mathbf{v}) =$

(f) $\nabla \times (\nabla \psi) =$

Problem 3

Assume $\mathbf{f} = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + f\hat{\mathbf{k}}$ and $\mathbf{v} = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}}$, where f, u, v, w are all scalar functions of x, y, z.

(a) What is $\nabla \cdot \mathbf{f}$?

(b) What is $\nabla \cdot \mathbf{v}$?

(c) What is $\nabla \times \mathbf{f}$?

(d) What is $\mathbf{f} \times \mathbf{v}$?