Coriolis Force

A **Coriolis parameter** is defined as

$$f_c = 2 \cdot \Omega \cdot \sin(\phi) \qquad \bullet(9.10)$$

where $2 \cdot \Omega = 1.458 \times 10^{-4}\ s^{-1}$, and ϕ is latitude. This parameter is constant at any fixed location. At mid-latitudes, the magnitude is of order of $f_c = 1 \times 10^{-4}\ s^{-1}$.

Coriolis force is an apparent force caused by the rotation of the earth. It acts perpendicular to the wind direction, to the right in the N. Hemisphere, and to the left in the Southern (Fig 9.3). In the N. Hemisphere, it is:

$$\frac{F_{x\ CF}}{m} = f_c \cdot V \qquad \bullet(9.11a)$$

$$\frac{F_{y\ CF}}{m} = -f_c \cdot U \qquad \bullet(9.11b)$$

Thus, there is no Coriolis force when there is no wind. Coriolis force cannot cause the wind to blow; it can only change its direction.

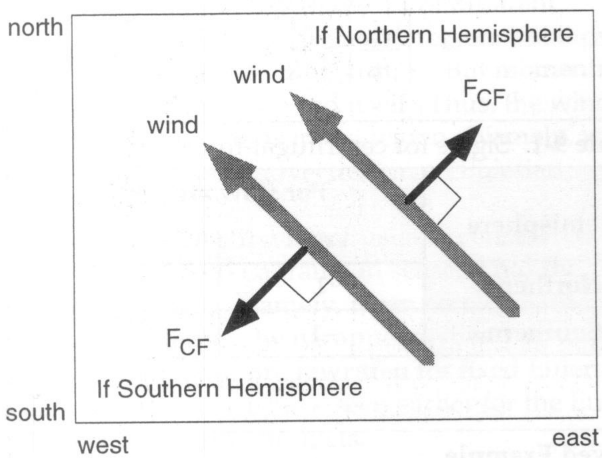


Figure 9.3
Coriolis force (dark lines).

Solved Example

Find the Coriolis force at Norman, Oklahoma, given a wind of $U = 10$ m/s.

Solution

Given: $U = 10$ m/s, $\phi = 35.2°$N at Norman.
Find: $F_{x\ CF}/m = ?\ m \cdot s^{-2}$.

First, find the Coriolis parameter using eq. (9.10):
$f_c = (1.458 \times 10^{-4}\ s^{-1}) \cdot \sin(35.2°) = 8.4 \times 10^{-5}\ s^{-1}$.

Coriolis force in the y-direction (eq. 9.11b) is:

$$\frac{F_{y\ CF}}{m} = -(8.4 \times 10^{-5} s^{-1})\left(10 \frac{m}{s}\right) = \underline{\mathbf{-8.4 \times 10^{-4}}}\ m \cdot s^{-2}.$$

Check: Units OK. Physics OK.
Discussion: The – sign means force is north to south.

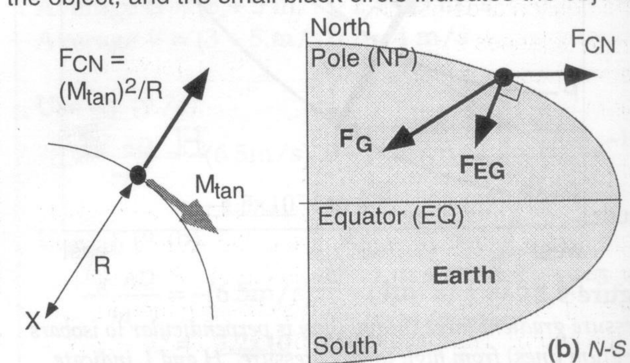
FOCUS • Coriolis Force

In 1835, Gaspar Gustave Coriolis used kinetic energy conservation to explain the apparent force that now bears his name. The following clarification was provided by Anders Persson in 1998.

Background

Coriolis force can be interpreted as the difference between two other forces: centrifugal force and gravitational force.

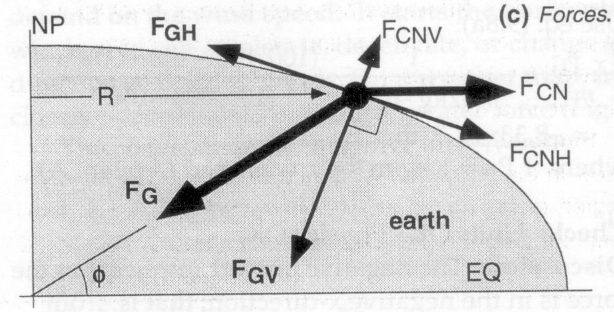
As discussed previously, **centrifugal force** is $F_{CN} / m = M_{tan}^2 / R$, where M_{tan} is the tangential velocity of an object moving along a curved path with radius of curvature R (see Fig a). This force increases if the object moves faster, or if the radius becomes smaller. The symbol X marks the center of rotation of the object, and the small black circle indicates the object.



(a) *Centrifugal force.* **(b)** *N-S slice through earth.*

Because the earth is plastic (i.e., deformable), centrifugal force due to the earth's rotation and **gravitational force** F_G have shaped the surface into an ellipsoid, not a sphere. This is exaggerated in Fig b. The vector sum of F_G and F_{CN} is the effective gravity, F_{EG}. This effective gravity acts perpendicular to the local surface, and defines the direction we call **down**. Thus, a stationary object feels no net force (the downward force is balanced by the earth holding it up).

Now that "up" and "down" are identified, we can split centrifugal and actual gravitational forces into local horizontal (subscript $_H$) and vertical (subscript $_V$) components (Fig c). Because F_{CN} is always parallel to the equator, trigonometry gives $F_{CNH} \cong F_{CN} \cdot \sin(\phi)$, where ϕ is latitude.



(c) *Forces.*

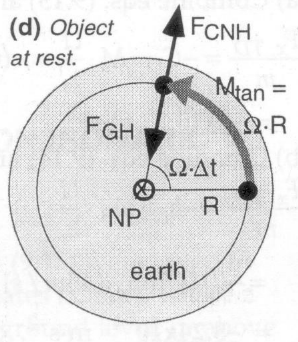
(continued next column)

FOCUS • Coriolis Force *(continuation)*

Objects at Rest

The earth rotates counterclockwise when viewed from above the north pole (NP). During time interval Δt, any single meridian (a longitude line, such as labeled with distance R in Fig d) will rotate by angle $\Omega\Delta t$, where Ω is the angular velocity of earth ($\cong$ 360°/24h).

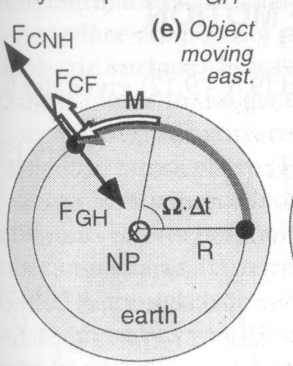
Suppose an object (the dark circle) is at rest on this meridian. Then during the same time interval Δt, it will move as shown by the gray arrow, at speed $M_{tan} = \Omega \cdot R$. Because this movement follows a parallel (latitude line), and parallels encircle the earth's axis, the stationary object is turning around a circle. This creates centrifugal force. The horizontal component F_{CNH} balances F_{GH}, giving zero net apparent force on the object (Fig d).



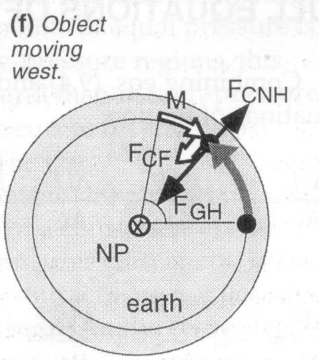
(d) *Object at rest.*

Objects Moving East or West

Next, we can ask what happens if the object moves eastward with velocity M relative to the earth's surface (shown by the thin white arrow in Fig e). The earth is rotating as before, as indicated by the thin meridian lines in the figure. Thus, the total movement of the object is faster than before, as shown by the gray arrow. This implies greater total centrifugal force, which results in a greater horizontal component F_{CNH}. However, the gravity component F_{GH} is unchanged.



(e) *Object moving east.*



(f) *Object moving west.*

Thus, those two forces (F_{CNH} & F_{GH}) do NOT balance. The difference between them is a net force to the right of the relative motion M. This force difference is called **Coriolis force**, F_{CF}, and is indicated by the thick white arrow in Fig e.

Similarly, for an object moving westward (thin white arrow in Fig f), the net tangential velocity (gray arrow) is slower, giving an imbalance between F_{CNH} & F_{GH} that acts to the right of M. This is identified as Coriolis force.

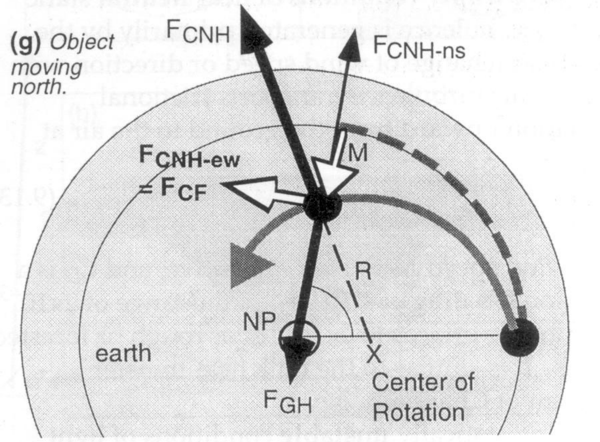
(continued next column)

FOCUS • Coriolis Force *(continuation)*

Objects Moving North or South

For a northward moving object, the rotation of the earth (dashed thick gray line) and the relative motion of the object (M, thin white arrow) combine to cause a path shown with the solid thick gray line (Fig g). This has a smaller radius of curvature (R) about a center of rotation (X) that is NOT on the north pole (o). The smaller radius causes a greater horizontal component of centrifugal force (F_{CNH}), which points outward from X.

We can conceptually divide this horizontal centrifugal force into a north-south component ($F_{CNH\text{-}ns}$) and an east-west component ($F_{CNH\text{-}ew}$). The southward component $F_{CNH\text{-}ns}$ balances the horizontal gravitational component F_{GH}, which hasn't changed. However, the east-west component is acting to the right of the relative object motion M, and is identified as Coriolis force $F_{CF} = F_{CNH\text{-}ew}$.



(g) *Object moving north.*

Similarly, a southward moving object has a larger radius of curvature, giving a Coriolis force to the right. In fact, an object moving in any arbitrary direction has Coriolis force acting to the right in the N. Hemisphere.

Magnitude of Coriolis Force

For an object at rest (Figs c & d), $F_{GH} = F_{CNH} \equiv F_{CNHR}$, where subscript $_R$ denotes "rest". At rest, $M_{tan\ rest} = \Omega \cdot R$. For an eastward moving object (Fig e), Coriolis force is defined as:

$$F_{CF} \equiv F_{CNH} - F_{GH} \qquad \text{(definition)}$$
$$= F_{CNH} - F_{CNHR} \qquad \text{(line 1 of this subsection)}$$
$$= \sin(\phi)\cdot[F_{CN} - F_{CNR}] \qquad \text{(from Fig c)}$$
$$F_{CF}/\,m = \sin(\phi)\cdot[(M_{tan})^2/R - (M_{tan\ rest})^2/R]$$
$$= \sin(\phi)\cdot[(\Omega\cdot R + M)^2/R - (\Omega\cdot R)^2/R]$$
$$= \sin(\phi)\cdot[(2\cdot\Omega\cdot M) + (M^2/R)]$$

But the last term is so small, it can be neglected compared to the first term. Thus:

$$F_{CF}/m \cong 2\cdot\Omega\cdot\sin(\phi)\cdot M$$
$$\equiv f_c\cdot M \qquad \text{(from eq. 9.10)}$$

The same answer is found for motion in any direction.