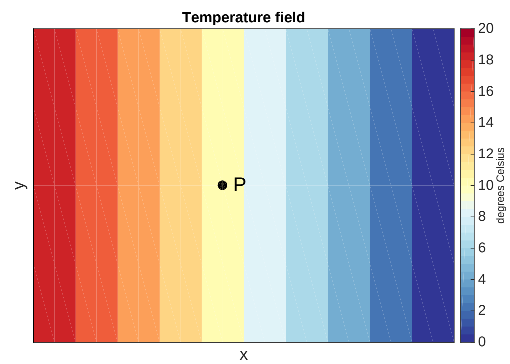
Homework 2

due Sept. 1, 2017

[50 pts.]

Name: ____

Problem 1: Lagrangian vs Eulerian perspectives



Consider a fluid (e.g. the atmosphere) flowing with steady (no time dependence) velocity $\mathbf{u} = (U, 0)$. Now, imagine that the fluid has a linear slope in temperature (T) in the x-direction of magnitude Λ.

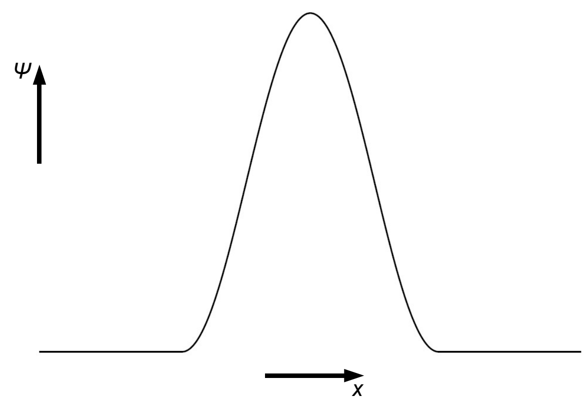
(a) What is the sign of Λ according to the figure?

(b) What is the ∇T, i.e. the gradient of T?

(c) What is $\frac{DT}{Dt}$ i.e. the material rate of change in temperature following a parcel assuming no heating sources or sinks?

(d) At a fixed point P, what is the time rate of change of temperature ($\frac{\partial T}{\partial t}$)? Are things getting warmer or colder at point P according to your equation? (Hint: use the definition of the material derivative).

Problem 2: Passive tracer advection



(a) Consider a one-dimensional passive tracer who's density is a function of x and t only: $\psi = \psi(x,t)$. Imagine a Gaussian-blob of this tracer (see figure) is placed in a steady flow field where $u(x,t) = U$, where U is a constant (i.e. not a function of time or space). Starting from the mass continuity equation, show that the equation for the density of ψ at time t and position x is given by the one-dimensional advection equation:

$$\frac{\partial \psi}{\partial t} + U\frac{\partial \psi}{\partial x} = 0 \tag{0.1}$$

(b) Show that this is equivalent to

$$\frac{D\psi}{Dt} = 0 \tag{0.2}$$

(c) The solution to the advection equation for $u(x,t) = U$ is

$$\psi(x,t) = \psi_0(x - Ut) \tag{0.3}$$

with initial condition

$$\psi(x, t = 0) = \psi_0(x). \tag{0.4}$$

Describe in words the evolution of the initial Gaussian blob using the solution as part of your explanation. Discuss the differences in the evolution of the tracer from the material and eulerian perspectives.

(d) Now consider the case where $u(x,t) = \Lambda|x|$, where Λ is a constant such that $\Lambda > 0$, and $|x|$ is the absolute value of x. Describe in words how the tracer blob will evolve in time. (Drawing some sketches might help). In this case, what form does the continuity equation take? How does it compare to Eq. (0.1) for a constant flow field?

(e) Test your intuition about the case where $u(x,t) = \Lambda|x|$ by numerically integrating your equation from part (d) above.

Step 1: Estimate the time tendencies and spatial derivatives with the following differencing schemes:

$$\frac{\partial \psi_j^i}{\partial t} \approx \frac{\psi_j^{i+1} - \psi_j^i}{\Delta t}, \tag{0.5}$$

$$\frac{\partial \psi_j^i}{\partial x} \approx \frac{\psi_j^i - \psi_{j-1}^i}{\Delta x} \tag{0.6}$$

where the superscript i is the time stamp, the subscript j is the spatial location, and Δt and Δx are the time step and grid spacing, respectively.

Step 2: Convert your equation from (d) into numerical form, and solve for the density ψ_j^{i+1} at a future time $i+1$ at location j.

Step 3: Write a loop in the programming language of your choice to compute ψ at each time step and grid box, and plot the distribution of the tracer density at time $t = 0, 2, 4, 6, ...22$ for $\Lambda = 0.10$, $\Delta t = 0.01$, and $\Delta x = 0.10$. For the initial gaussian pulse, let

$$\psi(x, t=0) = \psi_0(x) = e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{0.7}$$

where $\mu = 5.0$[1] and $\sigma = 1.0$.

[1]For fun, try $\mu = 1$ but keeping every other parameter the same. It looks pretty cool.