Homework 8

due Nov. 17, 2017

[50 pts.]

Problem 1: Internal gravity waves and buoyancy frequency

Gravity waves are ubiquitous in stably stratified fluids and arise from the restoring force of gravity. For this problem, you are going to utilize the Boussinesq system of equations (nearly incompressible, constant background density) to gain insight into *internal gravity waves*.

Consider a parcel of air displaced upward by an infinitesimal distance δz from its equilibrium level z_0 in the Boussinesq system. Assume that the pressure of the parcel remains the same as its environment, that is, neglect pressure perturbations.

(a) Derive the following equation for the position of the parcel with higher-order terms removed:

$$\ddot{z} \equiv \frac{Dw}{Dt} \approx -g\frac{\delta\rho}{\rho_0} \tag{0.1}$$

where ρ_0 describes a background state, with $\delta\rho$ representing small perturbations about ρ_0.

(b) Assume that $\delta\rho = \rho(z_0) - \rho(z_0 + \delta z)$ and use a Taylor expansion of ρ around z_0 to derive the following equation (remember to once again remove terms of higher order):

$$\ddot{z}' + N^2 z' \approx 0 \tag{0.2}$$

where

$$N^2 \equiv -g\frac{\partial_z \rho|_{z_0}}{\rho_0} \quad \text{and} \quad z' \equiv z - z_0 \tag{0.3}$$

(c) Solve the equation from (b) and physically interpret its solution. That is, distinguish between the cases of $N^2 > 0$, $N^2 = 0$ and $N^2 < 0$.

(d) Provide time series plots of the evolution of z' for these three cases. These can be rough sketches or computer generated plots.